Quantum Mechanics I/Physics IV

March 4,2009

Total Marks 100

Answer all question.

1.(a) Let H be a separable Hilbert space over C with orthogonal basis $\{e_j\}_1^\infty$. Show that the following are equivalent.

(i) $x = \sum_{j=1}^{\infty} x_j e_j$ with $x_j =< e_j, x >$

(ii) $\sum_1^\infty |x_j|^2 = \|x\|^2$, for $x \epsilon H$

(iii) $< x, y >= \sum_{j=1}^{\infty} \bar{x}_j y_j$ for x,y ϵH

(Remember that in the definition of the inner product, conjugate linearity is in the left entry.)

(b) Show that every separable Hilbert space is isometrically isomorphic to l_2.

2. Let A,B be bounded linear operators defined everywhere in H. Show that

(i) $\|AB\| \leq \|A\| \|B\|$,

(ii) $\|A^*\| = \|A\|$

(iii) if$A = A^*$,then$\|A\|=$ sup $\frac{|<Af,f>|}{\|f\|^2}$

(Hint:Prove and use the relation:

Re$< f, Ag >= \frac{1}{4}\{< (f+g), A(f+g) > - \langle (f-g), A(f-g) \rangle \})$

(iv) $\|A^*A\| = \|A\|^2$

3. (a) Let$\varphi, \psi : M \subseteq \mathbb{R}^{2d} \rightarrow \mathbb{R}$ be two smooth functions, where M is the phase-space of generalised positions and momenta $(q_j, p_j : 0 \leq j \leq d)$. Show that

$$\frac{d}{dt}\{\varphi, \psi\} = \Big\{ \{\varphi, \psi\}, H \Big\}$$

,where $\{.,.\}$ is the Poisson Bracket and H is the (smooth) Hamiltorian.

(b) For a particle with position and momentum q_j and p_j respectively $(1 \leq j \leq 3)$, define the angular momenta $L(L_i, 1 \leq i \leq 3)$ by $L_i = \epsilon_{ijk} q_j p_k$, where ϵ_{ijk} is the elementary completely antisymmetric teuser (=0 if any two of the three indices are equal, and $=^{+}_{-} 1$ according as (ijk) is an even or odd permutation of the symbols (123)). Using the Hamilton's equations for q and p, show that $\frac{dL}{dt} = \{L, H\}$ as a vector-equation.

(c) For a particle in a central potential V (from a fixed centre of force)in 3-dimensions ,derive the equation of motion for the angular momenta.